About this book

This Past Papers Workbook is designed to help you practice and prepare for your IB exams in the most effective and focused way possible.

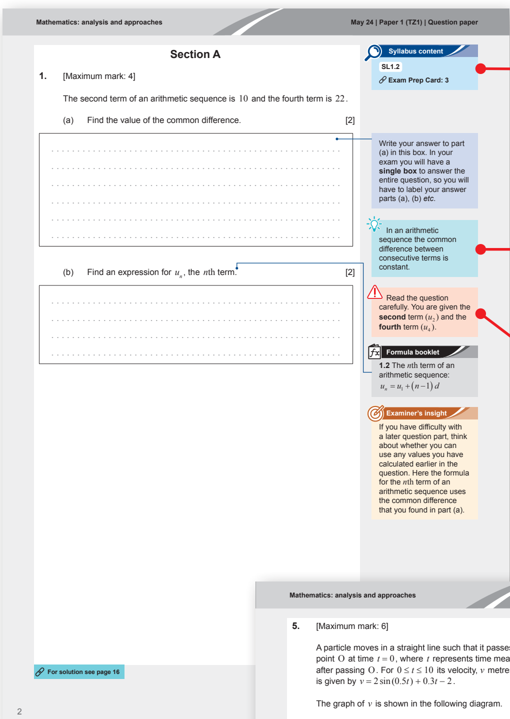
Unlock your exams with two full sets of official IB past papers, comprehensive worked solutions and easy-to-understand marking guidelines, all supported with hints and tips based on the official IB Subject Reports.

Examiner's insight

IB examiners publish **Subject Reports** after each exam session, highlighting common student mistakes and strategies for improvement. This workbook includes advice based on these reports—giving you expert guidance from the people who actually mark your papers!

Question papers

Use these features to maximize your past paper practice time, boost your confidence and master your exam technique.


Mathematics: analysis and approaches

May 24 | Paper 1 (TZ1) | Question paper

Section A

1. [Maximum mark: 4]

The second term of an arithmetic sequence is 10 and the fourth term is 22.

(a) Find the value of the common difference. [2]

(b) Find an expression for u_n, the nth term. [2]

Syllabus content SL1.2 Exam Prep Card: 3

Write your answer to part (a) in this box. In your exam you will have a **single box** to answer the entire question, so you will have to label your answer parts (a), (b) etc.

In an arithmetic sequence the common difference between consecutive terms is constant.

Read the question carefully. You are given the **second** term (u_2) and the **fourth** term (u_4).

Formula booklet: 1.2 The nth term of an arithmetic sequence: $u_n = u_1 + (n-1)d$

Examiner's insight: If you have difficulty with a later question part, think about whether you can use any values you have calculated earlier in the question. Here the formula for the nth term of an arithmetic sequence uses the common difference that you found in part (a).

For solution see page 16

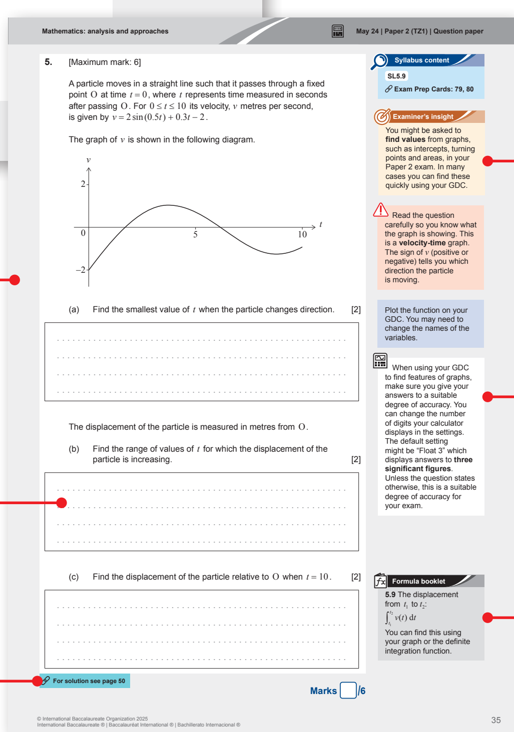
2

Syllabus content

Links to the relevant sections of the IB Subject Guide and the IB *Unlocked* Exam Prep Cards are provided with every question.

Hints with this icon remind you of a key piece of knowledge or key mathematical skill needed for this question.

Hints with this icon highlight common errors and misconceptions.


Mathematics: analysis and approaches

May 24 | Paper 2 (TZ1) | Question paper

5. [Maximum mark: 6]

A particle moves in a straight line such that it passes through a fixed point O at time $t = 0$, where t represents time measured in seconds after passing O. For $0 \le t \le 10$ its velocity, v metres per second, is given by $v = 2\sin(0.5t) + 0.3t - 2$.

The graph of v is shown in the following diagram.

(a) Find the smallest value of t when the particle changes direction. [2]

The displacement of the particle is measured in metres from O.

(b) Find the range of values of t for which the displacement of the particle is increasing. [2]

(c) Find the displacement of the particle relative to O when $t = 10$. [2]

Syllabus content SL5.9 Exam Prep Cards: 79, 80

Examiner's insight: You might be asked to **find values** from graphs, such as intercepts, turning points and areas. In your Paper 2 exam. In many cases you can find these quickly using your GDC.

Read the question carefully so you know what the graph is showing. This is a **velocity-time** graph. The sign of v (positive or negative) tells you which direction the particle is moving.

Plot the function on your GDC. You may need to change the names of the variables.

When using your GDC to find features of graphs, make sure you give your answers to a suitable degree of accuracy. You can change the number of digits your calculator displays in the settings. The default setting might be "Float 3" which displays answers to three **significant figures**. Unless the question states otherwise, this is a suitable degree of accuracy for your exam.

Formula booklet: 5.9 The displacement from t_1 to t_2: $\int_{t_1}^{t_2} v(t)\,dt$. You can find this using your graph or the definite integration function.

For solution see page 50

Marks /6

35

Complete official papers direct from the IB, so there are no surprises in your exam.

Write your answers directly on the page so you can keep everything in one place and stay organized.

Find solutions and marking guidelines quickly, mark your work accurately and track your progress.

Examiner's insight

Based on the responses of real students, these boxes give you advice about how to structure your answers, show your working clearly, and target all the available marks.

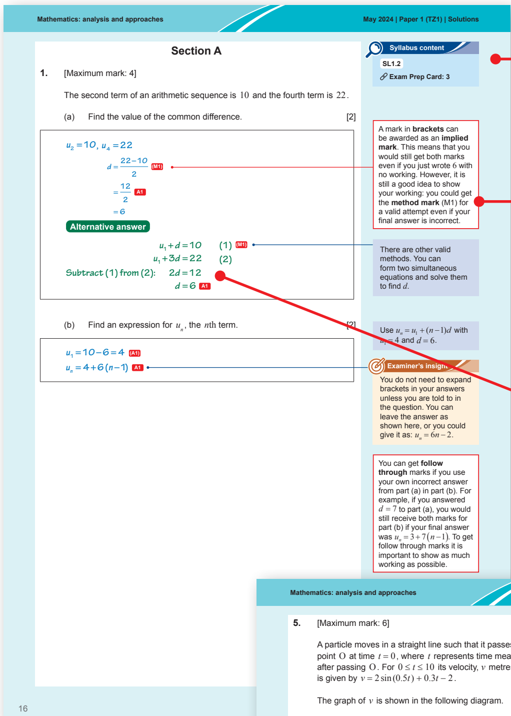
Hints with this icon show you where you can use your graphic display calculator (GDC) to answer questions quickly and save time in your exam.

Formula booklet

Using the formula booklet efficiently can save time and improve accuracy. These boxes show you which formulae are relevant for each question.

Solutions and marking guidelines

Mark your own work like an examiner with complete worked solutions and detailed marking guidelines for every question.

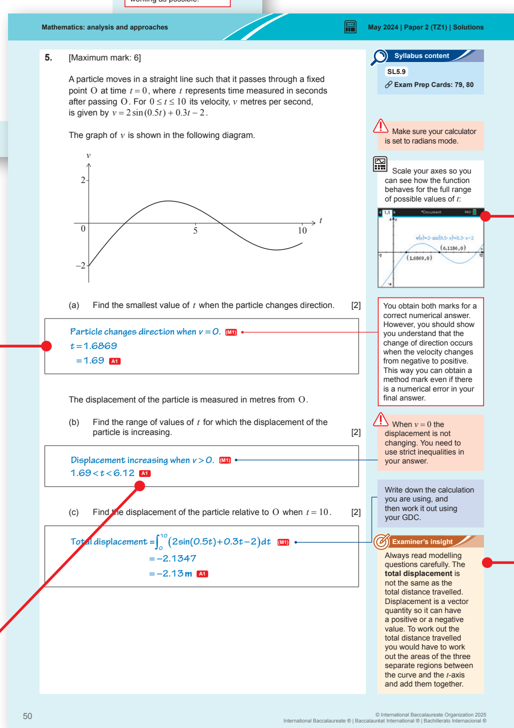


Look for the blue background to find the solution pages easily.

Marking hints next to every question help you understand mark allocations so you can score your own work quickly and accurately.

Alternative answer

Some questions can be approached in more than one way. Look out for this heading for alternative methods that would also be awarded full marks.



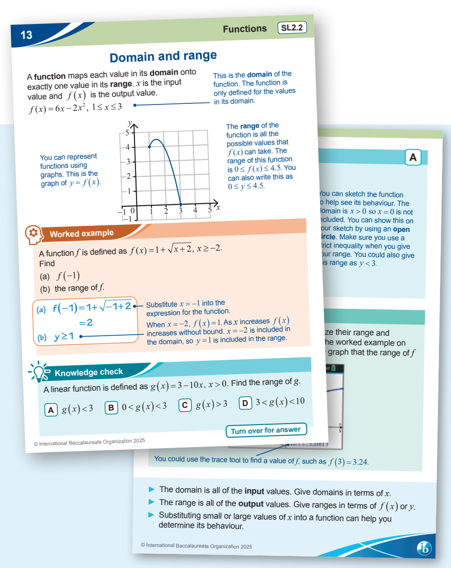
Screenshots show you how the working for this question might appear on a typical GDC screen. Make sure you are familiar with your own GDC and know how to put it in exam mode.

Improve your exam technique by seeing exactly what a top student would write.

Examiner's insight

Information on how this question was answered by other students in the actual exam provides even more support. Gain insight into common misconceptions and learn strategies to improve your own answers.

Different types of marks are clearly shown, and are positioned next to the relevant part of the answer. See exactly how marks would be awarded in the exam.



Boost your skills and fill in any knowledge gaps with the **IB *Unlocked* Exam Prep Cards.**

- Complete, concise coverage of every syllabus topic
- Practice questions on every card with detailed solutions
- Exam focused hints, technology tips and insights from examiners
- Links to relevant cards from every question in this workbook

How your exam is marked

Understanding how your exam is marked helps you to get every mark you deserve. It will help you plan your answers better, show your working more clearly and avoid simple mistakes that can cost you marks.

There are **three** basic types of mark available in your exam:

M Method marks

These marks are awarded for planning and attempting a strategy to solve the problem.

- Always show your working, even when you use your GDC
- Begin each step of working on a new line, and don't skip any steps
- Write clearly and legibly, and do not rush

A Answer marks

These marks are awarded for finding correct intermediate and final answers.

- Check your working for simple arithmetic errors
- Make sure your final answer is rounded to the correct degree of accuracy
- Read the question carefully and give all the information you are asked for

R Reasoning marks

These marks are awarded for showing correct reasoning or justification.

- Make your mathematical thinking visible to the examiner
- Use correct mathematical language and notation
- Use words as well as symbols to justify your answer

An outlier is a value that is less than $Q_1 - 1.5 \times \text{IQR}$ or greater than $Q_3 + 1.5 \times \text{IQR}$.

(b) Show that 11.7 is an outlier. [3]

$Q_3 + 1.5 \times \text{IQR} = 5.7 + 1.5 \times 3.8$ (M1)

$= 11.4$ (A1)

$11.7 > 11.4$ so 11.7 is an outlier R1

This **method mark** is awarded for attempting to use the rule given in the question to find the upper bound for an outlier. You can obtain this mark for writing down the calculation even if you make a mistake when you work it out.

This **answer mark** is awarded for correctly calculating the upper bound. Your answer needs to match this one in order to obtain this mark.

To obtain the **reasoning mark** you need to give mathematical evidence to show that 11.7 is an outlier. You can do this by comparing 11.7 with the upper bound you calculated using an inequality.

- Think carefully before **crossing out** work. Crossed-out work won't be marked unless you specifically ask for it to be considered.
- Do not round **intermediate values** in your working. Write down as many of the digits from your GDC display as possible, or use exact values. In the solutions in this workbook, intermediate values have generally been truncated to five significant figures, and final answers have been rounded to three significant figures where appropriate.
- If the question asks for a specific **degree of accuracy** make sure you use that for your final answer. Otherwise, give exact answers or round to three significant figures.
- Check your answers **make sense**. For example, probabilities must be between 0 and 1, angles in a triangle must be less than 180°, and answers in practical contexts must be reasonable.

Implied marks

Implied marks are marks shown in brackets, like (M1) or (A1). These are marks that can be awarded without this line of working being explicitly seen. You can only be awarded implied marks if the subsequent answer or line of working is exactly right.

You should not rely on implied marks. It is much safer to show each step of your working clearly. That way you can be awarded these marks even if there is an error somewhere in your final answer.

Solve $3m^2 + 5m - 2 = 0$

$(3m-1)(m+2)=0$ (M1) (A1)

$m=\frac{1}{3}$ or $m=-2$ A1

These are implied marks. The (M1) mark is for making an attempt to factorize the quadratic (or use the quadratic formula), and the (A1) mark is for doing this correctly.

Here are three attempts to answer this question:

Solve $3m^2 + 5m - 2 = 0$

$m=\frac{1}{3}$ or $m=-2$

This student has obtained the correct answer so the first two marks can be implied. This student scores all **3 marks**.

Solve $3m^2 + 5m - 2 = 0$

$m=1$ or $m=-2$

This student has not given a fully correct answer and has shown no working. This student would obtain **0 marks**.

Solve $3m^2 + 5m - 2 = 0$

$(3m-1)(m+2)=0$

$m=1$ or $m=-2$

This student has made the same mistake but has shown their working. This student would obtain the first **2 marks**.

Follow through marks

Sometimes you need to use your answer from one part of a question in later parts of the same question. If you calculate an incorrect answer in part (a), you can still obtain full marks in parts (b), (c) and so on, as long as you use **your answer correctly**.

This is an attempt to answer the question shown on the opposite page, using an incorrect value for the IQR.

(a) For these data, find the interquartile range. [2]

IQR $= 4.9 - 1.9$ (A1)

$= 3.0$

This student has used an incorrect value of Q_3 when calculating the IQR. They have used the correct value for Q_1 so they still obtain **1 mark** for showing their working.

An outlier is a value that is less than $Q_1 - 1.5 \times \text{IQR}$ or greater than $Q_3 + 1.5 \times \text{IQR}$.

(b) Show that 11.7 is an outlier. [3]

$Q_3 + 1.5 \times \text{IQR} = 4.9 + 1.5 \times 3.0$ (M1)

$= 9.4$ (A1)

$11.7 > 9.4$ so 11.7 is an outlier R1

The student has used **their incorrect values** of Q_3 and the IQR in this formula. However, they have used the formula correctly and have carried out their calculation accurately. The conclusion is still valid for their incorrect values so they obtain all **3 marks** for this question part.

Progress tracker and grade boundaries

Use this table to record your scores on each paper. The actual grade boundaries from each paper are given in the tables so you can track your progress towards your exam goals.

Time zone 1: Paper 1 My score ☐ / 80 My grade ☐

Score	0–8	9–17	18–25	26–35	36–48	49–58	59–80
Grade	1	2	3	4	5	6	7

Time zone 1: Paper 2 My score ☐ / 80 My grade ☐

Score	0–7	8–16	17–24	25–35	36–48	49–58	59–80
Grade	1	2	3	4	5	6	7

Time zone 2: Paper 1 My score ☐ / 80 My grade ☐

Score	0–5	6–11	12–20	21–31	32–44	45–57	58–80
Grade	1	2	3	4	5	6	7

Time zone 2: Paper 2 My score ☐ / 80 My grade ☐

Score	0–7	8–13	14–22	23–33	34–45	46–58	59–80
Grade	1	2	3	4	5	6	7

Effective exam preparation with past papers

Here are some strategies for making the most of your Past Papers Workbook.

- **Use the first set of papers (Time Zone 1) for skill building:** do not time yourself, and keep your textbooks, notes and Exam Prep Cards available to build your confidence and hone your exam technique.
- **Always attempt before checking solutions:** mark each question individually or complete whole papers first, but never look at the solution until you have attempted the question.
- **Master every solution:** make sure you fully understand the solutions and can identify any errors you made in your own working.
- **Target your exam preparation:** use the syllabus references and links to the Exam Prep Cards to re-visit the topics you struggled with, then retry those questions.
- **Attempt the second set of papers (Time Zone 2) under exam conditions:** time yourself and put your notes away—you can always refer back to them when you are marking your work.

Examiner's insight

The **formula booklet** and your **GDC** are two of the most powerful tools in your exam:

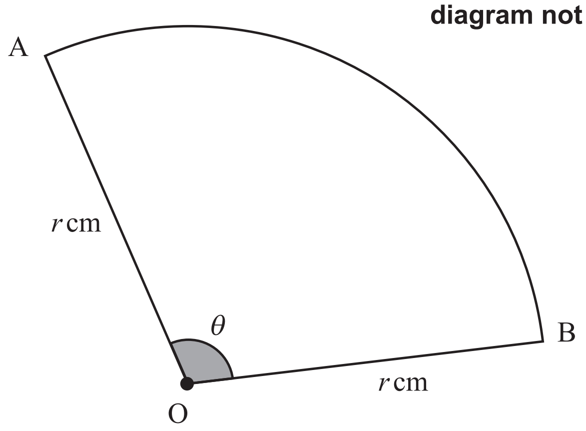
- Use the formula booklet **throughout your course**—not just in the run up to the exam.
- Make sure you know how your GDC works, and how to put it into **exam mode**.
- **Show your working** even when you use your GDC—write down any expressions, equations or values you enter, and state the specific functions you are using.

4. [Maximum mark: 8]

Points A and B lie on the circumference of a circle of radius $r\,\mathrm{cm}$ with centre at O.

The sector OAB is shown on the following diagram. The angle $\mathrm{A\hat{O}B}$ is denoted as θ and is measured in radians.

diagram not to scale



The perimeter of the sector is $10\,\mathrm{cm}$ and the area of the sector is $6.25\,\mathrm{cm}^2$.

(a) Show that $4r^2 - 20r + 25 = 0$. [4]

(b) Hence, or otherwise, find the value of r and the value of θ. [4]

For solution see page 19

Marks /8

Syllabus content

SL3.4

Exam Prep Card: 36

Formula booklet

3.4 For a sector with radius r and angle θ measured in radians:
Length of arc: $l = r\theta$
Area of sector: $A = \frac{1}{2}r^2\theta$

Use these two facts to write two equations involving r and θ. The equation you want to find involves r only, so you need to eliminate θ. Rearrange the resulting equation to obtain the quadratic equation given.

The **perimeter** is given, not the arc length.

Examiner's insight

"**Show that**" means all the marks are available for your working. Make sure you show your method clearly. You can also use additional words in your answer to explain your processes.

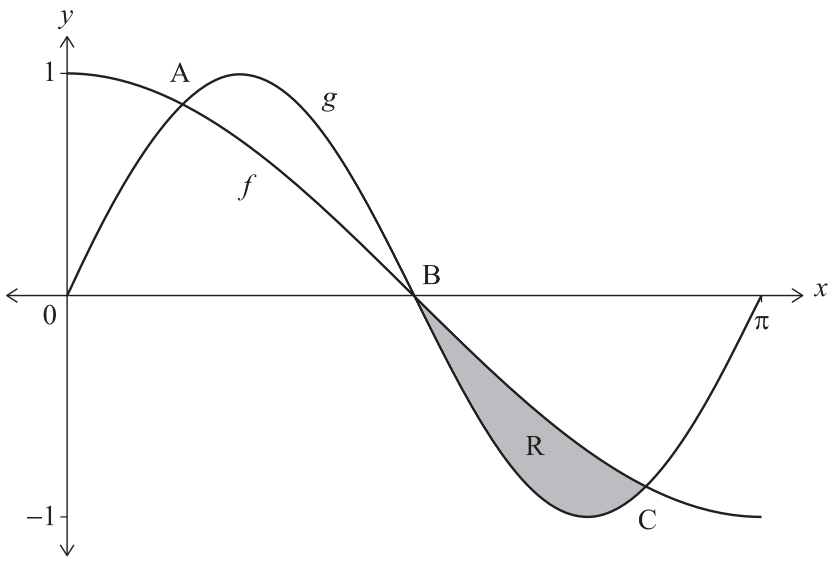
A common error in "show that …" questions is to work backwards from the answer. Begin with the information given in the question. $4r^2 - 20r + 25 = 0$ should be the **last** line of your working, not the first line.

You have been given an equation involving r in part (a). Even if you have difficulty with part (a), you can still solve this quadratic equation to find r in part (b).

5. [Maximum mark: 7]

Consider the functions $f(x) = \cos x$ and $g(x) = \sin 2x$, where $0 \le x \le \pi$.

The graph of f intersects the graph of g at the point A, the point $\mathrm{B}\left(\frac{\pi}{2}, 0\right)$ and the point C as shown on the following diagram.



(a) Find the x-coordinate of point A and the x-coordinate of point C. [3]

The shaded region R is enclosed by the graph of f and the graph of g between the points B and C.

(b) Find the area of R. [4]

For solution see page 20

Marks /7

Syllabus content

SL3.8 SL5.10 SL5.11

Exam Prep Cards: 43, 78

The solutions to the equation $f(x) = g(x)$ represent the x-coordinates of the points where the curves $y = f(x)$ and $y = g(x)$ intersect.

Formula booklet

3.6 Double angle identity: $\sin 2\theta = 2 \sin \theta \cos \theta$

The values of x are measured in **radians**. Make sure you give your answers in radians, not in degrees.

You can use definite integration to find the area between a curve and the x-axis. To find the area **between** $y = f(x)$ and $y = g(x)$ you can use either of these methods:

- integrate $f(x) - g(x)$ between appropriate limits
- integrate $f(x)$ and $g(x)$ separately and subtract one from the other.

Examiner's insight

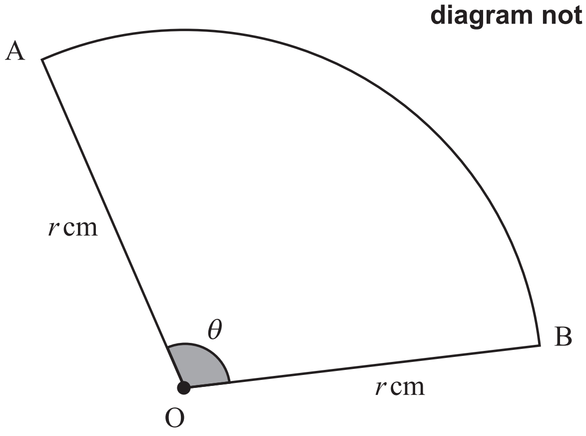
Remember this is a **non-calculator** problem. Make sure you know the **exact values** of common trigonometric ratios and that you can work with them confidently.

4. [Maximum mark: 8]

Points A and B lie on the circumference of a circle of radius r cm with centre at O.

The sector OAB is shown on the following diagram. The angle $\mathrm{A\hat{O}B}$ is denoted as θ and is measured in radians.

diagram not to scale



The perimeter of the sector is 10 cm and the area of the sector is $6.25\ \text{cm}^2$.

(a) Show that $4r^2 - 20r + 25 = 0$. [4]

$2r + r\theta = 10$ (1) A1

$\frac{1}{2}r^2\theta = 6.25$ (2) A1

Rearrange (1): $r(2+\theta) = 10$

$\theta = \frac{10}{r} - 2$ M1

Substitue into (2): $\frac{1}{2}r^2\left(\frac{10}{r} - 2\right) = 6.25$ A1

$2r^2\left(\frac{10}{r} - 2\right) = 25$

$20r - 4r^2 = 25$ So $4r^2 - 20r + 25 = 0$

(b) Hence, or otherwise, find the value of r and the value of θ. [4]

$4r^2 - 20r + 25 = 0$

$(2r-5)^2 = 0$ (M1)

$r = \frac{5}{2}$ A1

$\theta = \frac{10}{r} - 2 = \frac{10}{\left(\frac{5}{2}\right)} - 2$ (M1)

$= 10\left(\frac{2}{5}\right) - 2 = 4 - 2$

$= 2$ A1

Syllabus content

SL3.4

Exam Prep Card: 36

Label your equations (1) and (2). Use these numbers to refer to these equations in your working.

The perimeter of the sector is made of **two radii** and the **arc**.

Examiner's insight

There are **no implied marks** in this "show that…" question. The examiner needs to see all of the steps shown here to award full marks:

- equation based on perimeter
- equation based on area
- attempt to eliminate θ
- correct equation in r only.

The required equation is in r only, so you need to eliminate θ. A substitution method is shown. You could also use elimination by multiplying equation (1) by r and multiplying equation (2) by 2.

At the end of your answer it is a good idea to write down what you are trying to show. You do not get a mark for this, as it is given in the question. In a markscheme this would have AG next to it, to indicate that this answer is given in the question.

You can write this as a fraction or as a decimal.

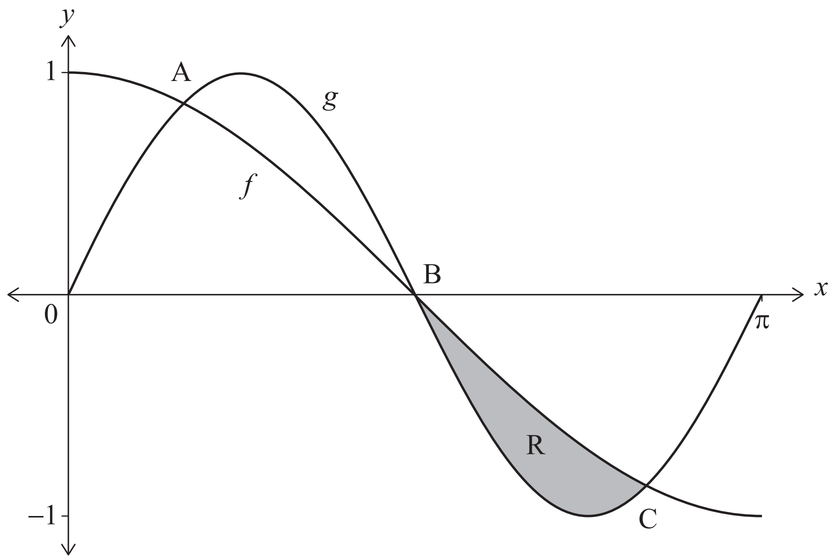
Substitute into one of your equations to find the value of θ.

Remember that θ is measured in radians.

5. [Maximum mark: 7]

Consider the functions $f(x)=\cos x$ and $g(x)=\sin 2x$, where $0 \le x \le \pi$.

The graph of f intersects the graph of g at the point A, the point $\mathrm{B}\left(\frac{\pi}{2}, 0\right)$ and the point C as shown on the following diagram.



(a) Find the x-coordinate of point A and the x-coordinate of point C. [3]

$\cos x = \sin 2x$

$\cos x = 2\sin x \cos x$ **(M1)**

$1 = 2\sin x$

$\sin x = \frac{1}{2}$ **(A1)**

$x = \frac{\pi}{6}, \frac{5\pi}{6}$ **A1**

The shaded region R is enclosed by the graph of f and the graph of g between the points B and C.

(b) Find the area of R. [4]

$$\int_{\frac{\pi}{2}}^{\frac{5\pi}{6}} (\cos x - \sin 2x)\,dx = \left[\sin x + \frac{1}{2}\cos 2x\right]_{\frac{\pi}{2}}^{\frac{5\pi}{6}}$$ **(M1)** **A1**

$$= \left(\sin\frac{5\pi}{6} + \frac{1}{2}\cos\frac{5\pi}{3}\right) - \left(\sin\frac{\pi}{2} + \frac{1}{2}\cos\pi\right)$$ **M1**

$$= \left(\frac{1}{2} + \frac{1}{4}\right) - \left(1 - \frac{1}{2}\right)$$

$$= \frac{1}{4}$$ **A1**

Syllabus content

SL3.8 SL5.10 SL5.11

Exam Prep Cards: 43, 78

You can obtain this mark by showing that you used the double angle identity for $\sin 2x$.

This step is valid because $\cos x \ne 0$ at points A and C. This approach would not allow you to find the point of intersection at B.

Examiner's insight

Write down all the possible solutions in the given domain. The domains of the functions are given in radians, and the horizontal axis of the graph is labelled in radians. Make sure that you give your answer in radians in terms of π. If you gave your answer as $x = 30°, 150°$ you would only obtain two of the possible three marks.

You can use either of these methods to integrate $\sin 2x$:

$$\int \sin 2x\,\mathrm{d}x = -\frac{1}{2}\cos 2x + C$$

or

$$\int 2\sin x \cos x\,\mathrm{d}x = \sin^2 x + C$$

The first method is shown here.

For part (b) only the first method mark can be implied. To obtain all four marks you need to show:

- a correct integration
- substitution of the limits
- the correct final area of R.

You can obtain full marks for part (b) if you work in degrees, as long as you show your working and obtain the correct area.