About this book

This Past Papers Workbook is designed to help you practice and prepare for your IB exams in the most effective and focused way possible.

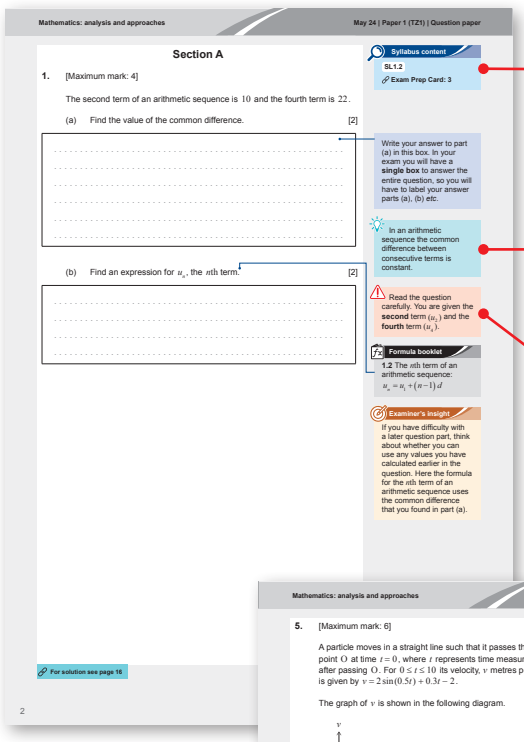
Unlock your exams with two full sets of official IB past papers, comprehensive worked solutions and easy-to-understand marking guidelines, all supported with hints and tips based on the official IB Subject Reports.

Examiner's insight

IB examiners publish **Subject Reports** after each exam session, highlighting common student mistakes and strategies for improvement. This workbook includes advice based on these reports—giving you expert guidance from the people who actually mark your papers!

Question papers

Use these features to maximize your past paper practice time, boost your confidence and master your exam technique.

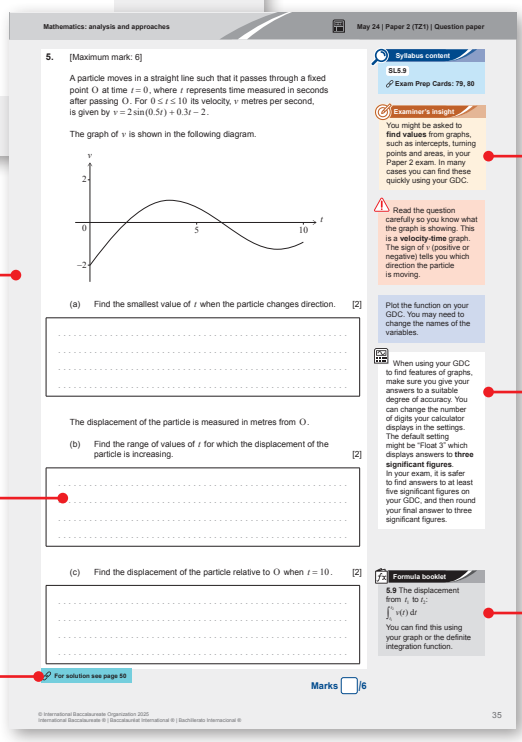


Syllabus content

Links to the relevant sections of the IB Subject Guide and the IB *Unlocked* Exam Prep Cards are provided with every question.

Hints with this icon remind you of a key piece of knowledge or key mathematical skill needed for this question.

Hints with this icon highlight common errors and misconceptions.



Complete official papers direct from the IB, so there are no surprises in your exam.

Write your answers directly on the page so you can keep everything in one place and stay organized.

Find solutions and marking guidelines quickly, mark your work accurately and track your progress.

Examiner's insight

Based on the responses of real students, these boxes give you advice about how to structure your answers, show your working clearly, and target all the available marks.

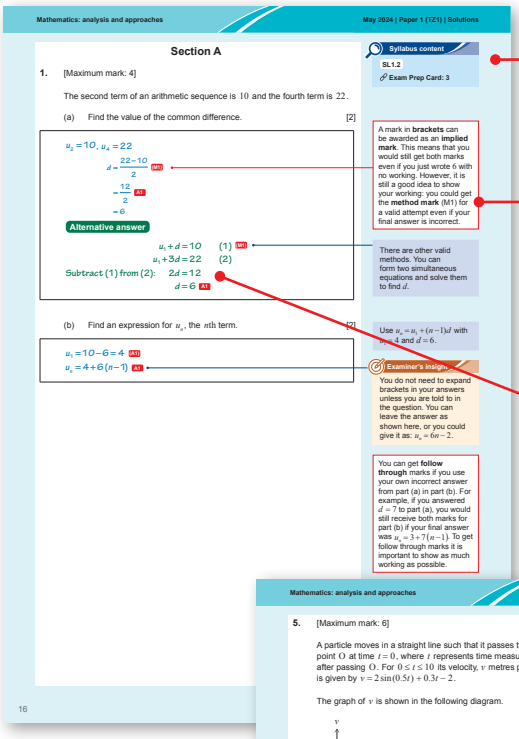
Hints with this icon show you where you can use your graphic display calculator (GDC) to answer questions quickly and save time in your exam.

Formula booklet

Using the formula booklet efficiently can save time and improve accuracy. These boxes show you which formulae are relevant for each question.

Solutions and marking guidelines

Mark your own work like an examiner with complete worked solutions and detailed marking guidelines for every question.

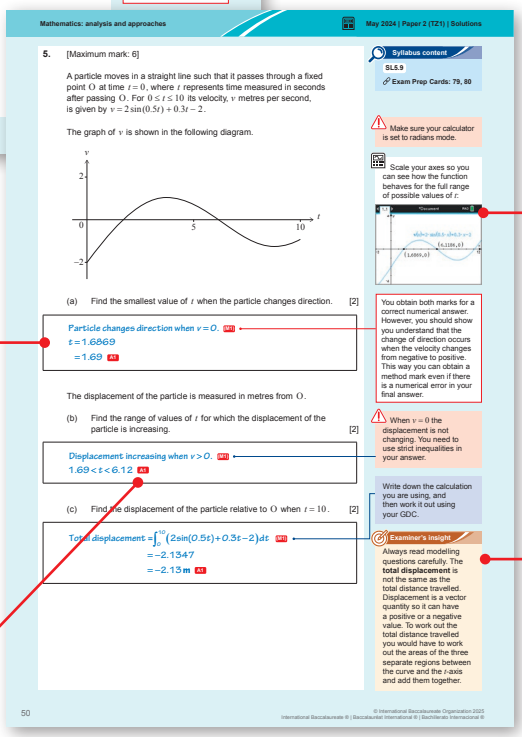


Look for the blue background to find the solution pages easily.

Marking hints next to every question help you understand mark allocations so you can score your own work quickly and accurately.

Alternative answer

Some questions can be approached in more than one way. Look out for this heading for alternative methods that would also be awarded full marks.



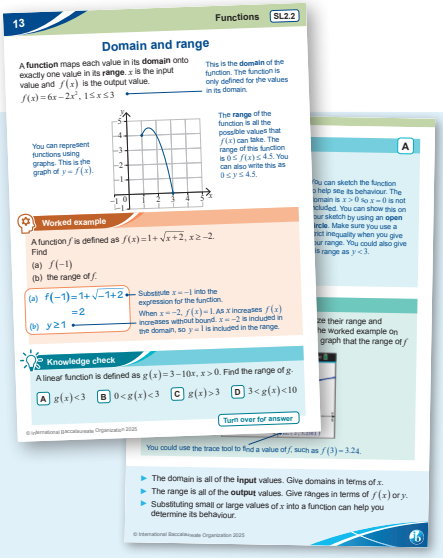
Improve your exam technique by seeing exactly what a top student would write.

Different types of marks are clearly shown, and are positioned next to the relevant part of the answer. See exactly how marks would be awarded in the exam.

Screenshots show you how the working for this question might appear on a typical GDC screen. Make sure you are familiar with your own GDC and know how to put it in exam mode.

Examiner's insight

Information on how this question was answered by other students in the actual exam provides even more support. Gain insight into common misconceptions and learn strategies to improve your own answers.



Boost your skills and fill in any knowledge gaps with the **IB *Unlocked* Exam Prep Cards**.

- Complete, concise coverage of every syllabus topic
- Practice questions on every card with detailed solutions
- Exam focused hints, technology tips and insights from examiners
- Links to relevant cards from every question in this workbook

How your exam is marked

Understanding how your exam is marked helps you to get every mark you deserve. It will help you plan your answers better, show your working more clearly and avoid simple mistakes that can cost you marks.

There are **three** basic types of mark available in your exam:

M Method marks

These marks are awarded for planning and attempting a strategy to solve the problem.

- Always show your working, even when you use your GDC
- Begin each step of working on a new line, and don't skip any steps
- Write clearly and legibly, and do not rush

A Answer marks

These marks are awarded for finding correct intermediate and final answers.

- Check your working for simple arithmetic errors
- Make sure your final answer is rounded to the correct degree of accuracy
- Read the question carefully and give all the information you are asked for

R Reasoning marks

These marks are awarded for showing correct reasoning or justification.

- Make your mathematical thinking visible to the examiner
- Use correct mathematical language and notation
- Use words as well as symbols to justify your answer

An outlier is a value that is less than $Q_1 - 1.5 \times \text{IQR}$ or greater than $Q_3 + 1.5 \times \text{IQR}$.

(b) Show that 11.7 is an outlier. [3]

$Q_3 + 1.5 \times \text{IQR} = 5.7 + 1.5 \times 3.8$ (M1)

$= 11.4$ (A1)

$11.7 > 11.4$ so 11.7 is an outlier R1

This **method mark** is awarded for attempting to use the rule given in the question to find the upper bound for an outlier. You can obtain this mark for writing down the calculation even if you make a mistake when you work it out.

This **answer mark** is awarded for correctly calculating the upper bound. Your answer needs to match this one in order to obtain this mark.

To obtain the **reasoning mark** you need to give mathematical evidence to show that 11.7 is an outlier. You can do this by comparing 11.7 with the upper bound you calculated using an inequality.

- Think carefully before **crossing out** work. Crossed-out work won't be marked unless you specifically ask for it to be considered.
- Do not round **intermediate values** in your working. Write down as many of the digits from your GDC display as possible, or use exact values. In the solutions in this workbook, intermediate values have generally been truncated to five significant figures, and final answers have been rounded to three significant figures where appropriate.
- If the question asks for a specific **degree of accuracy** make sure you use that for your final answer. Otherwise, give exact answers or round to three significant figures.
- Check your answers **make sense**. For example, probabilities must be between 0 and 1, angles in a triangle must be less than 180°, and answers in practical contexts must be reasonable.

Implied marks

Implied marks are marks shown in brackets, like (M1) or (A1). These are marks that can be awarded without this line of working being explicitly seen. You can only be awarded implied marks if the subsequent answer or line of working is exactly right.

You should not rely on implied marks. It is much safer to show each step of your working clearly. That way you can be awarded these marks even if there is an error somewhere in your final answer.

Solve $3m^2 + 5m - 2 = 0$

$(3m-1)(m+2)=0$ (M1) (A1)

$m=\frac{1}{3}$ or $m=-2$ A1

These are implied marks. The (M1) mark is for making an attempt to factorize the quadratic (or use the quadratic formula), and the (A1) mark is for doing this correctly.

Here are three attempts to answer this question:

Solve $3m^2 + 5m - 2 = 0$

$m=\frac{1}{3}$ or $m=-2$

This student has obtained the correct answer so the first two marks can be implied. This student scores all **3 marks**.

Solve $3m^2 + 5m - 2 = 0$

$m=1$ or $m=-2$

This student has not given a fully correct answer and has shown no working. This student would obtain **0 marks**.

Solve $3m^2 + 5m - 2 = 0$

$(3m-1)(m+2)=0$

$m=1$ or $m=-2$

This student has made the same mistake but has shown their working. This student would obtain the first **2 marks**.

Follow through marks

Sometimes you need to use your answer from one part of a question in later parts of the same question. If you calculate an incorrect answer in part (a), you can still obtain full marks in parts (b), (c) and so on, as long as you use **your answer correctly**.

This is an attempt to answer the question shown on the opposite page, using an incorrect value for the IQR.

(a) For these data, find the interquartile range. [2]

$\text{IQR} = 4.9 - 1.9$ (A1)

$= 3.0$

This student has used an incorrect value of Q_3 when calculating the IQR. They have used the correct value for Q_1 so they still obtain **1 mark** for showing their working.

An outlier is a value that is less than $Q_1 - 1.5 \times \text{IQR}$ or greater than $Q_3 + 1.5 \times \text{IQR}$.

(b) Show that 11.7 is an outlier. [3]

$Q_3 + 1.5 \times \text{IQR} = 4.9 + 1.5 \times 3.0$ (M1)

$= 9.4$ (A1)

$11.7 > 9.4$ so 11.7 is an outlier R1

The student has used **their incorrect values** of Q_3 and the IQR in this formula. However, they have used the formula correctly and have carried out their calculation accurately. The conclusion is still valid for their incorrect values so they obtain all **3 marks** for this question part.

Progress tracker and grade boundaries

Use this table to record your scores on each paper. The actual grade boundaries from each paper are given in the tables so you can track your progress towards your exam goals.

Time zone 1: Paper 1 My score ☐ / 80 My grade ☐

Score	0–11	12–19	20–29	30–38	39–49	50–57	58–80
Grade	1	2	3	4	5	6	7

Time zone 1: Paper 2 My score ☐ / 80 My grade ☐

Score	0–10	11–20	21–29	30–40	41–52	53–62	63–80
Grade	1	2	3	4	5	6	7

Time zone 2: Paper 1 My score ☐ / 80 My grade ☐

Score	0–10	11–19	20–26	27–36	37–46	47–58	59–80
Grade	1	2	3	4	5	6	7

Time zone 2: Paper 2 My score ☐ / 80 My grade ☐

Score	0–9	10–18	19–25	26–35	36–46	47–59	60–80
Grade	1	2	3	4	5	6	7

Effective exam preparation with past papers

Here are some strategies for making the most of your Past Papers Workbook.

- **Use the first set of papers (Time Zone 1) for skill building:** do not time yourself, and keep your textbooks, notes and Exam Prep Cards available to build your confidence and hone your exam technique.
- **Always attempt before checking solutions:** mark each question individually or complete whole papers first, but never look at the solution until you have attempted the question.
- **Master every solution:** make sure you fully understand the solutions and can identify any errors you made in your own working.
- **Target your exam preparation:** use the syllabus references and links to the Exam Prep Cards to re-visit the topics you struggled with, then retry those questions.
- **Attempt the second set of papers (Time Zone 2) under exam conditions:** time yourself and put your notes away—you can always refer back to them when you are marking your work.

Examiner's insight

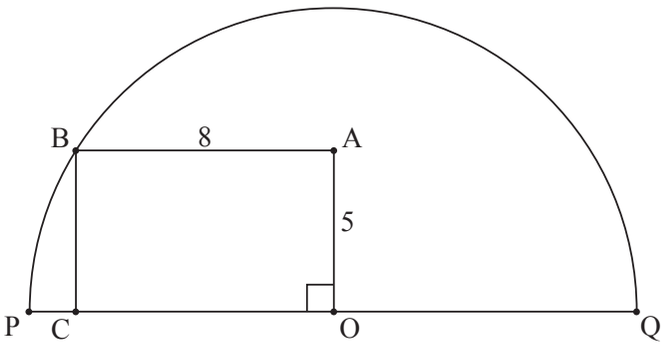
The **formula booklet** and your **GDC** are two of the most powerful tools in your exam:

- Use the formula booklet **throughout your course**—not just in the run up to the exam.
- Make sure you know how your GDC works, and how to put it into **exam mode**.
- **Show your working** even when you use your GDC—write down any expressions, equations or values you enter, and state the specific functions you are using.

11. [Maximum mark: 5]

The following diagram shows a semicircle with centre O and diameter PQ. A rectangle OABC is also shown, such that $AB = 8$ and $OA = 5$.

diagram not to scale



Find the length of the arc BQ. [5]

For solution see page 36

Marks /5

Syllabus content

SL3.3 SL3.4

Exam Prep Cards: 27, 33

You need to know how to find a length using Pythagoras' theorem and how to find an angle using trigonometry. You also need to be familiar with how to use the formula for the arc length of a circle.

Begin by marking what you are trying to find on the diagram and then see if there is any missing information that you need. For example, here you will need the radius which is [BO] and either angle BOQ or BOP. You can then find the arc length.

Angle BOC is not equal to angle BOA.

Formula booklet

3.4 Length of an arc:

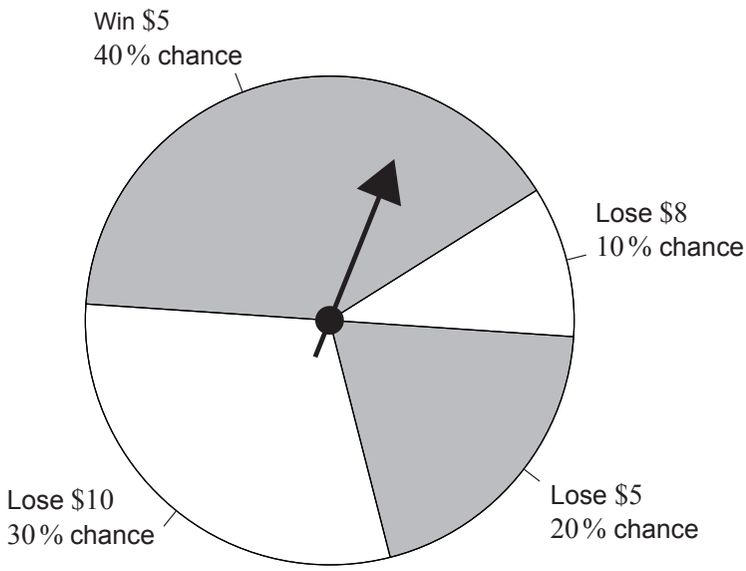
$$l = \frac{\theta}{360} \times 2\pi r$$

where θ is the angle measured in degrees and r is the radius.

12. [Maximum mark: 8]

Zac raises funds for a library by running a game where players spin a needle. The final position of the needle results in an outcome where a player wins or loses money. The outcomes, with associated probabilities, are shown in the following diagram.

diagram not to scale



Let X represent the amount that a player of this game wins.

(a) (i) Find the expected value of X.

(ii) Interpret your answer to part (a)(i). [3]

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(Question 12 continues on the next page)

For solution see page 37

Syllabus content

SL1.2 SL4.7

Exam Prep Card: 52

The expected value is a number, not a probability.

In a fair game, nobody has an advantage. You will not gain or lose money in the long term, so $\mathrm{E}(X) = 0$.

Formula booklet

4.7 The expected value of a discrete random variable X:

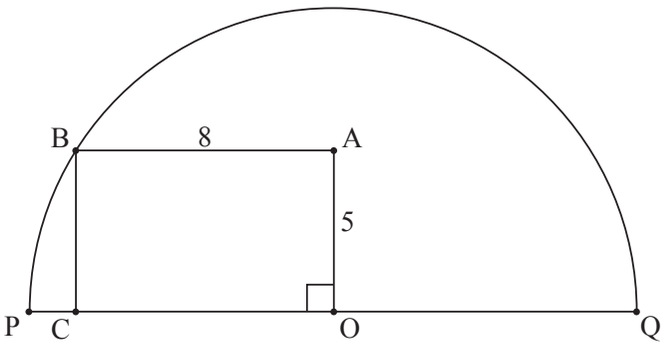
$$\mathrm{E}(X) = \sum_{i=1}^{k} x_i \mathrm{P}(X = x_i)$$

For part (a)(ii), think about what an expected value is and refer to the result you found in part (a)(i).

11. [Maximum mark: 5]

The following diagram shows a semicircle with centre O and diameter PQ. A rectangle $OABC$ is also shown, such that $AB = 8$ and $OA = 5$.

diagram not to scale



Find the length of the arc BQ. [5]

OB is a radius

$OB = \sqrt{5^2 + 8^2} = \sqrt{89}$ (A1)

$\tan B\hat{O}A = \left(\frac{8}{5}\right)$

$\tan^{-1}\left(\frac{8}{5}\right) = B\hat{O}A = 57.995$ (A1)

So $B\hat{O}Q = 90 + B\hat{O}A = 147.995$ (M1)

$\text{arc } BQ = \frac{147.995}{360} \times 2\pi \times \sqrt{89}$ (M1)

$= 24.4 \text{ m}$ A1

Alternative answer

OB is a radius

$OB = \sqrt{5^2 + 8^2} = \sqrt{89}$ (A1)

$\tan B\hat{O}P = \left(\frac{5}{8}\right)$

$\tan^{-1}\left(\frac{5}{8}\right) = B\hat{O}P = 32.005$ (A1)

$\text{arc } BP = \frac{32.005}{360} \times 2\pi \times \sqrt{89} = 5.2697$ (M1)

$\text{arc } PQ = \pi \times \sqrt{89} = 29.638$

$\text{arc } PQ - \text{arc } BP = 29.638 - 5.2697$ (M1)

$= 24.4 \text{ m}$ A1

Syllabus content

SL3.3 SL3.4

Exam Prep Cards: 27, 33

In questions like this you may find it useful to annotate the diagram to help you keep track of what you are trying to find.

OB is a radius of the semicircle. Find this first and you can use it to find either angle BOQ or angle BOP. You can then use your angle to find the arc length.

Examiner's insight

There are a lot of implied marks in this question which means that all 5 marks can be awarded if the final answer (here 24.4) is correct. However, if the final answer is incorrect and no working is seen, no marks can be awarded. Try to write down as much working as you can to let the examiner know what you are trying to do.

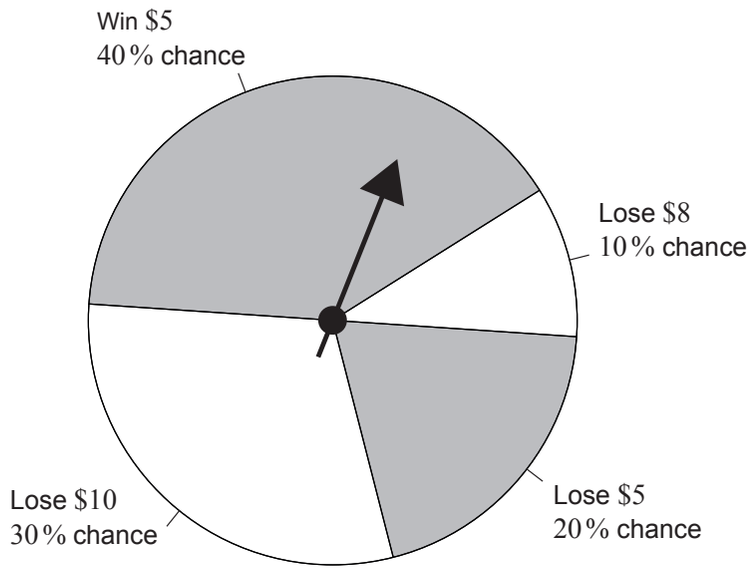
These are the values you have found previously, substituted into $l = \frac{\theta}{360} \times 2\pi r$

Arc PQ is the length of a semicircle. Once you know this length you can then subtract arc BP from it.

12. [Maximum mark: 8]

Zac raises funds for a library by running a game where players spin a needle. The final position of the needle results in an outcome where a player wins or loses money. The outcomes, with associated probabilities, are shown in the following diagram.

diagram not to scale



Let X represent the amount that a player of this game wins.

(a) (i) Find the expected value of X.

(ii) Interpret your answer to part (a)(i). [3]

(i) $E(X) = 5 \times 0.40 - 8 \times 0.1 - 5 \times 0.2 - 10 \times 0.3$ (M1)

$E(X) = -2.8$ A1

(ii) *On average, players will lose $2.80 per game.* A1

(Question 12 continues on the next page)

Syllabus content

SL1.2 SL4.7

Exam Prep Card: 52

Examiner's insight

In part (a)(i), be careful not to confuse the expected value of X with the probability of winning a game (which here is $40\,\%$).

Examiner's insight

In part (a)(ii), you must make reference to the result being an **expected** average rather than something that will definitely happen. For example, you might have said "Players can expect to lose $\$2.80$ on average" but you would not get the mark if you said "Players will lose $\$2.80$".